Problem 6.2

$I = V_{emf}/R$. Since the single-turn loop is not moving or changing shape with time, $V_{emf}^m = 0\,V$ and $V_{emf} = V_{emf}^{tr}$. Therefore, from Eq. (6.8),

$$I = V_{emf}^{tr}/R = \frac{-1}{R}\int_S \frac{\partial B}{\partial t} \cdot ds$$

If we take the surface normal to be $+\hat{z}$, then the right hand rule gives positive flowing current to be in the $+\hat{\phi}$ direction.

$$I = \frac{-A}{R}\frac{\partial}{\partial t} B_0 \sin \omega t = -\frac{AB_0\omega}{R}\cos\omega t \text{ (A)},$$

where A is the area of the loop.

(a) A, ω and R are positive quantities. At $t=0$, $\cos\omega t = 1$ so $I < 0$ and the current is flowing in the $\boxed{-\hat{\phi} \text{ direction}}$ (so as to produce ~~and~~ an induced magnetic field that opposes B)

(b) At $\omega t = \pi/4$, $\cos\omega t = \sqrt{2}/2$ so $I < 0$ and the current is still flowing in the $\boxed{-\hat{\phi} \text{ direction.}}$

(c) At $\omega t = \pi/2$, $\cos\omega t = 0$ so $I = 0$. There is $\boxed{\text{no current}}$ flowing in either direction.

Problem 6.6

(a) The magnetic field due to the wire is

$$B = \hat{\phi}\frac{\mu_0 I}{2\pi r} = -\hat{x}\frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\phi} = -\hat{x}$ and $r = y$.

The flux passing through the loop is

$$\phi = \int_S B \cdot ds = \int_{5\text{cm}}^{15\text{cm}} \left(-\hat{x}\frac{\mu_0 I}{2\pi y}\right) \cdot [-\hat{x}10(\text{cm})]\,dy$$

$$= \frac{\mu_0 I \times 10^{-1}}{2\pi} \ln\frac{15}{5}$$

$$= \frac{4\pi \times 10^{-7} \times 5\cos(2\pi \times 10^4 t) \times 10^{-1}}{2\pi} \times 1.1$$

$$= 1.1 \times 10^{-7} \cos(2\pi \times 10^4 t) \quad (\text{Wb})$$

$$V_{emf} = -\frac{d\phi}{dt} = 1.1 \times 2\pi \times 10^4 \sin(2\pi \times 10^4 t) \times 10^{-7}$$

$$\boxed{= 6.9 \times 10^{-3} \sin(2\pi \times 10^4 t)\ (\text{V})}$$

(b) $I_{ind} = \frac{V_{emf}}{4+1} = \frac{6.9 \times 10^{-3}}{5} \sin(2\pi \times 10^4 t) = \boxed{1.38 \sin(2\pi \times 10^4 t)\ (\text{mA})}$

At $t=0$, B is a maximum, it points in $-\hat{x}$-direction, and since it varies as $\cos(2\pi \times 10^4 t)$, it is decreasing. Hence, the induced current has to be $\boxed{\text{CCW}}$ when looking down on the loop, as shown in the figure.

Problem 6.11

The surface of the cylinder has velocity u given by

$$u = \hat{\phi}\,\omega r = \hat{\phi}\,2\pi \times \cancel{\frac{1200}{60}} \times 5 \times 10^{-2} = \hat{\phi}\,2\pi \text{ (m/s)}.$$

$$V_{12} = \int_0^{l} (u \times B) \cdot dl = \int_0^{0.1} (\hat{\phi}\,2\pi \times \hat{r}\,6) \cdot \hat{z}\,dz = \boxed{-3.77 \text{ (V)}}$$

Problem 6.14

Since the voltage is of the form given by Eq. (6.46) with $V_0 = 30$ V and $\omega = 2\pi \times 10^6$ rad/s, the displacement current is given by Eq. (6.49):

$$I_d = -\frac{\varepsilon A}{d} V_0 \omega \sin \omega t$$

$$= -\frac{4 \times 8.854 \times 10^{-12} \times 10 \times 10^{-4}}{2 \times 10^{-2}} \times 30 \times 2\pi \times 10^6 \sin(2\pi \times 10^6 t)$$

$$= \boxed{-0.33 \sin(2\pi \times 10^6 t) \text{ (mA)}}$$

Problem 6.16

(a) $R = \dfrac{d}{\sigma A}$, $I_c = \dfrac{V}{R} = \dfrac{V\sigma A}{d}$

(b) $E = \dfrac{V}{d}$, $I_d = \dfrac{\partial D}{\partial t} \cdot A = \varepsilon A \dfrac{\partial E}{\partial t} = \dfrac{\varepsilon A}{d} \partial V/\partial t$

(c) The conduction curent is directly proportional to V, as characteristic of a resistor, whereas the displacement current varies as $\partial V/\partial t$, which is characteristic of a capacitor. Hence,

$$R = d/\sigma A \quad \text{and} \quad C = \varepsilon A/d$$

(d) $R = \dfrac{0.5 \times 10^{-2}}{2.5 \times 4 \times 10^{-4}} = 5\,\Omega$

Problem 6.20

Based on the given information, $\mathbf{J} = \hat{r} J_r(r)$

With $J_\phi = J_z = 0$, in cylindrical coordinates the divergence is given by $\nabla \cdot \mathbf{J} = \frac{1}{r}\frac{\partial}{\partial r}(r J_r)$

From Eq (6.54) $\nabla \cdot \mathbf{J} = -\partial \rho_v / \partial t = -\frac{\partial}{\partial t}(\rho_0 r \cos\omega t) = \rho_0 r \omega \sin\omega t$

Hence
$$\frac{1}{r}\frac{\partial}{\partial r}(r J_r) = \rho_0 r \omega \sin\omega t$$
$$\frac{\partial}{\partial r}(r J_r) = \rho_0 r^2 \omega \sin\omega t$$
$$\int_0^r \frac{\partial}{\partial r}(r J_r)\,dr = \rho_0 \omega \sin\omega t \int_0^r r^2\,dr$$
$$r J_r \Big|_0^r = (\rho_0 \omega \sin\omega t)\frac{r^3}{3}\Big|_0^r$$
$$J_r = \frac{\rho_0 \omega r^2}{3}\sin\omega t$$
and
$$\mathbf{J} = \hat{r} J_r = \boxed{\hat{r}\frac{\rho_0 \omega r^2}{3}\sin\omega t \quad (\mathrm{A/m^2})}$$

Problem 6.22

Converting to phasor form, the electric field is given by $\tilde{E}(z) = \hat{x}4e^{-j2z} - j\hat{y}3e^{-j2z}$ (V/m)

which can be used with Eq (6.87) to find the magnetic field:

$$\tilde{H}(z) = \frac{1}{-j\omega\mu} \nabla \times \tilde{E}$$

$$= \frac{1}{-j\omega\mu} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 4e^{-j2z} & -j3e^{-j2z} & 0 \end{vmatrix}$$

$$= \frac{1}{-j\omega\mu} (\hat{x}6e^{-j2z} - \hat{y}j8e^{-j2z})$$

$$= \frac{j}{6\times10^8 \times 4\pi \times 10^{-7}} (\hat{x}6 - \hat{y}j8)e^{-j2z}$$

$$= j\hat{x}\,8.0e^{-j2z} + \hat{y}\,10.6e^{-j2z} \text{ (mA/m)}$$

Converting back to instantaneous values, this is

$$\boxed{H(t, z) = -\hat{x}\,8.0\sin(6\times10^8 t - 2z) + \hat{y}\,10.6\cos(6\times10^8 t - 2z) \text{ (mA/m)}}$$